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The curvaton as a pseudo-Nambu-Goldstone boson

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Abstract: The field responsible for the cosmological curvature perturbations generated during a stage of primordial inflation might be the "curvaton", a field different from the inflaton field. To keep the effective mass of the curvaton small enough compared to the Hubble rate during inflation one may not invoke supersymmetry since the latter is broken by the vacuum energy density. In this paper we propose the idea that the curvaton is a pseudo Nambu-Goldstone boson (PNGB) so that its potential and mass vanish in the limit of unbroken symmetry. We give a general framework within which PNGB curvaton candidates should be explored. Then we explore various possibilities, including the case where the curvaton can be identified with the extra-component of a gauge field in a compactified five-dimensional theory (a Wilson line), where it comes from a Little-Higgs mechanism, and where it is a string axion so that supersymmetry is essential.

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JHEP07(2003)053

Contents

1. Introduction **1**

2. General estimates **3**
- 2.1 The curvaton potential 3
- 2.2 The curvature perturbation 4
- 2.3 The epoch of curvaton decay 5
- 2.4 The assumption of randomization 6
- 2.5 Inequalities following from the randomization assumption 7
- 2.6 Implementing the estimates 8

3. The symmetry broken by non-renormalizable terms **8**

4. The curvaton as a Wilson line **9**

5. The curvaton as a Little Higgs **13**

6. The curvaton as a string axion **14**
- 6.1 Onset of curvaton oscillations before radiation domination ($H_{\rm REH} < m$) 16
- 6.2 Onset of curvaton oscillations after radiation domination ($H_{\rm REH} \geq m$) 17

7. Conclusion **19**

JHEP07(2003)053

1. Introduction

It is now clear that structure in the Universe comes primarily from an almost scale-invariant superhorizon curvature perturbation [1]. This perturbation originates presumably from the vacuum fluctuation, during almost-exponential inflation, of some field with mass much less than the Hubble parameter H. Indeed, every such field acquires a nearly scale-invariant classical perturbation. The question is, which field is responsible for the curvature perturbation?

With two exceptions [2, 3], the universal assumption until 2001 was that the responsible field was the inflaton, defined in this context as the field whose value determines the end of inflation. Then it was proposed instead [4] that the responsible field is some 'curvaton' 'field different from the inflaton. The curvaton field oscillates during some radiation-dominated

era, causing its energy density to grow to at least 1% of the total and thereby generating the curvature perturbation. The curvaton hypothesis has since been the subject of a lot of attention [5]–[28] because it opens up new possibilities both for theory and observation.[1]

The requirement that the effective curvaton mass be much less than the Hubble parameter during inflation is a severe constraint. In this respect the situation for the curvaton is the same as that for the inflaton in the inflaton scenario.[2] In contrast with the inflaton case though, the effective curvaton mass should preferably also be much less than H even after inflation, since otherwise the curvaton perturbation has non-trivial evolution and one has to be careful that its perturbation is not driven to zero [25, 28].

To keep the effective mass of the inflaton or curvaton small enough, it seems natural to invoke supersymmetry (SUSY). Flat directions are frequent in supersymmetric theories, and they are stable under radiative corrections, as long as SUSY is not broken. However, in the early Universe SUSY is broken by the energy density, and as a result the effective mass-squared of each scalar field generically receives a contribution of order $\pm H^2$, both during [31, 32, 33] and after [34, 35] inflation.

An alternative possibility for keeping the effective mass sufficiently small is to make the inflaton or the curvaton a pseudo Nambu-Goldstone boson (PNGB), so that its potential vanishes in the limit where the corresponding global symmetry is unbroken. Then the effective mass-squared of the curvaton or the inflaton (defined as the second derivative of its potential) vanishes in the limit of unbroken symmetry and can indeed be kept small by keeping the breaking sufficiently small.

However, in the case of the inflaton the vanishing of its potential in the limit of unbroken symmetry raises two complications. First, one has to ensure that the entire scalar field potential does not vanish in the limit of unbroken symmetry. This means that one has to work with a hybrid inflation model, where the potential of some 'waterfall' field different from the inflaton is nonzero during inflation. Second, one has to ensure that the slope V' of the inflaton potential is not too small during inflation, since otherwise the curvature perturbation is too big. In particular, in the regime $V' \lesssim H^3$ the quantum fluctuation of the inflaton field would dominate its classical motion, leading to a curvature perturbation of order one ('eternal' inflation). For these and other reasons, it turns out that models making the inflaton a PNGB are fairly complicated, whether in the original context of the symmetry associated with a supergravity potential of the no-scale type [33, 36, 37, 38] or in the more recent context [39]–[43] of using a symmetry such as U(1), SU(N) or SO(N).

In contrast, the vanishing of the curvaton potential in the limit of exact symmetry raises no problems at all. In particular, it is quite all right for the slope of the curvaton potential to be so small that the entire range of the curvaton field lies in the quantum regime [4]. In this paper we explore in detail the idea that the curvaton is a PNGB. We consider the simplest case that the symmetry is a U(1), and look at schemes where SUSY is

[1] In a variant of the curvaton scenario [13, 14] the curvaton field only indirectly causes the growth of the curvature perturbation. Recently, a completely different scheme has been proposed [29, 30] in which the responsible field acts by perturbing the inflaton decay rate.

[2] In the curvaton scenario, the effective inflaton mass during inflation might be of order the Hubble parameter [12].

JHEP07(2003)053

optional as well as the string axion case where it is essential. The layout of the paper is as follows. In section 2 we establish some general formulas. In section 3 we recall the case [4] that the symmetry is broken by a non-renormalizable term. In section 4 we consider the case that the symmetry is broken by a Wilson line in an extra dimension, and in section 5 we consider the little-Higgs mechanism. (These cases for the inflaton have recently been considered in references [41, 42, 43].) In section 6 we consider the case that the curvaton is a string axion, and we conclude in section 7.

2. General estimates

2.1 The curvaton potential

We assume that the curvaton field σ is the PNGB of a broken U(1) symmetry, with some finite range $-\pi v < \sigma < \pi v$. Without loss of generality we can take $\sigma = 0$ to be the minimum of the potential. Keeping only one term in the power series the potential is then

$$V(\sigma) = (vm)^2 \left(1 - \cos\left(\frac{\sigma}{v}\right)\right) \tag{2.1}$$

$$\simeq \frac{1}{2} m^2 \sigma^2 \,, \tag{2.2}$$

where the second line is valid for small $\sigma \ll v$ so that m is the curvaton mass. Additional terms in the power series will modify the potential in some region around the maximum, but we assume that this region is small enough that it can be ignored. Because we are dealing with a PNGB, we shall in general assume that this potential is not significantly modified in the early Universe.

The curvaton scenario assumes almost-exponential inflation, with some Hubble parameter H_*. To avoid excessive gravitational wave production one needs $H_* \lesssim 10^{-5} M_{\rm P}$. We shall go further, and assume that inflation involves a slowly-rolling inflaton field. Then the requirement that the inflaton contribution to the curvature perturbation is some fraction x of the total gives [12]

$$\frac{H_*}{M_{\rm P}} \lesssim 10^{-5} x \,. \tag{2.3}$$

The curvaton scenario requires $x \ll 1$. The requirement that the curvaton potential be negligible during inflation corresponds to

$$vm \ll M_{\rm P} H_* \,. \tag{2.4}$$

It is assumed that the curvaton is light during inflation,

$$m \ll H_* \,, \tag{2.5}$$

so that on super-horizon scales it has a classical perturbation with an almost flat spectrum given by

$$\mathcal{P}_{\delta\sigma}^{1/2} = \frac{H_*}{2\pi} \,. \tag{2.6}$$

JHEP07(2003)053

2.2 The curvature perturbation

After horizon exit, the unperturbed curvaton field σ and its perturbation $\delta\sigma$ evolve according to the classical equations

$$\ddot{\sigma} + 3H\dot{\sigma} + V'(\sigma) = 0 \tag{2.7}$$

$$\ddot{\delta\sigma} + 3H\dot{\delta\sigma} + V''(\sigma)\delta\sigma = 0\,, \tag{2.8}$$

where the primes denote derivatives with respect to σ. Because of the flatness condition eq. (2.5) on the potential, the changes in σ and $\delta\sigma$ are negligible until $H \sim m$.

When $H \sim m$, the field starts to oscillate around zero. At this stage the curvaton energy density is $\rho_\sigma = \frac{1}{2}m^2\sigma_*^2$ while the total is $\rho \sim H^2 M_{\rm P}^2$. The fraction is therefore

$$\left.\frac{\rho_\sigma}{\rho}\right|_{H=m} \sim \left(\frac{\sigma_*}{M_{\rm P}}\right)^2. \tag{2.9}$$

The fraction is small provided that $\sigma_* \ll M_{\rm P}$, and we shall see in a moment that this condition is equivalent to the requirement that the inflaton gives only a small contribution to the curvature perturbation.

After a few Hubble times the oscillation will be sinusoidal except for the Hubble damping. The energy density ρ_σ will then be proportional to the square of the oscillation amplitude, and will scale like the inverse of the locally-defined comoving volume corresponding to matter domination. On the spatially flat slicing, corresponding to uniform local expansion, its perturbation has a constant value

$$\frac{\delta\rho_\sigma}{\rho_\sigma} = 2q\left(\frac{\delta\sigma}{\sigma}\right)_*. \tag{2.10}$$

The factor q accounts for the evolution of the field from the time that m/H becomes significant, and will be close to 1 provided that σ_* is not too close to the maximum value πv.[3]

The curvature perturbation ζ is supposed to be negligible when the curvaton starts to oscillate, growing during some radiation-dominated era when $\rho_\sigma/\rho \propto a$. After the curvaton decays ζ becomes constant. In the approximation that the curvaton decays instantly (and setting $q = 1$) it is then given by [9]

$$\zeta \simeq \frac{2r}{3}\left(\frac{\delta\sigma}{\sigma}\right)_*, \tag{2.11}$$

where

$$r \equiv \left.\frac{\rho_\sigma}{\rho}\right|_{\rm DEC}, \tag{2.12}$$

and the subscript DEC denotes the epoch of decay. The corresponding spectrum is

$$\mathcal{P}_\zeta^{\frac{1}{2}} \simeq \frac{2r}{3}\frac{H_*}{2\pi\sigma_*}. \tag{2.13}$$

[3] The correction factor is estimated in [44]. The factor q in principle also includes evolution of the field before the oscillation starts, but as already mentioned this evolution will be negligible when the curvaton is a PNGB. If instead the curvaton is not a PNGB, q may be very different from 1 [25, 28]

JHEP07(2003)053

It must match the observed value [45, 46] $\mathcal{P}_\zeta^{1/2} = 5 \times 10^{-5}$, which means that

$$\frac{H_*}{\sigma_*} \simeq \frac{5 \times 10^{-4}}{r}\,. \tag{2.14}$$

The current WMAP bound on non-gaussianity [47] requires $r > 9 \times 10^{-3}$. Using eqs. (2.3) and (2.14) we find

$$\frac{\sigma_*}{M_{\rm P}} \simeq 10^{-2} r^{-1} x \lesssim x\,. \tag{2.15}$$

As advertised, this is small because the inflaton is supposed to contribute a negligible fraction of the observed curvature perturbation.

2.3 The epoch of curvaton decay

The curvaton scenario can lead to non-gaussianity, and also to isocurvature perturbations which are generated by the curvaton field and hence are fully correlated with the curvature perturbation (residual isocurvature perturbations). Whether these are possible depends on the fraction r, that the curvaton contributes to the energy density by the time that it decays. The regime $r < 10^{-2}$ is forbidden, because it gives non-gaussianity above the level permitted by present WMAP data. In the regime $10^{-2} < r \ll 1$ there is significant non-gaussianity, which can be detected by by the PLANCK satellite or even by future WMAP data. On the other hand, isocurvature matter perturbations in this regime are either zero (if the matter is created after curvaton decay) or else bigger than is allowed by observation.

In the regime $r \simeq 1$ the situation is reversed. Non-gaussianity will be only at the second-order level, as in the inflaton scenario [49], and difficult to observe in the foreseeable future. On the other hand, isocurvature matter density perturbations can be generated at a level that will be detectable in the foreseeable future.

In order to evaluate r, one needs the decay rate Γ of the curvaton which determines the epoch $H \sim \Gamma$ of curvaton decay. One also needs to know the behaviour of the scale factor while the curvaton is oscillating. Here, we shall consider the two simplest cases. First, that the non-curvaton energy density corresponds to radiation throughout the oscillation. Second, that the non-curvaton energy density corresponds to matter until some 'reheating' epoch, after which it again corresponds to radiation. (In the simplest version of this second case, the matter domination era prior to the 'reheating' epoch will begin right after inflation.) In both cases, we use eq. (2.9), along with the fact that matter energy density (including the energy density of the oscillating curvaton) scales like a^{-3} while radiation energy density scales like a^{-4}, with the scale factor being $a \propto t^{\frac{2}{3}}$ during matter domination and $a \propto t^{\frac{1}{2}}$ during radiation domination. We find that at the decay epoch, the fraction r of the energy density due to the curvaton is given by

$$\frac{m\sigma_*^4}{\Gamma M_{\rm P}^4} \min\left\{1, \frac{H_{\rm REH}}{m}\right\} \sim \begin{cases} r^2 & (r \ll 1) \\ (1-r)^{-\frac{3}{2}} & (1-r \ll 1) \end{cases}\,, \tag{2.16}$$

JHEP07(2003)053

where $H_{\rm REH}$ is the Hubble parameter at reheating. Using eq. (2.14) and $r < 1$ this leads to the bound

$$\frac{H_*}{M_{\rm P}} \gtrsim 5 \times 10^{-4} \left(\frac{\Gamma}{m}\right)^{1/4} . \tag{2.17}$$

Since the curvaton should not disturb big bang nucleosynthesis (BBN) we require that its decay occurs earlier, which introduces the bound

$$\Gamma > H_{\rm BBN} = \frac{\pi}{3}\left(\frac{g_{\rm BBN}}{10}\right)^{1/2} \frac{T_{\rm BBN}^2}{M_{\rm P}} \simeq 4.5 \times 10^{-25}\,{\rm GeV}\,, \tag{2.18}$$

where $T_{\rm BBN} \simeq 1\,{\rm MeV}$ and $g_{\rm BBN} = 10.75$ is the relativistic degrees of freedom at the time. If the decay rate is of gravitational strength, corresponding to $\Gamma \sim m^3/M_{\rm P}^2$, the above bound demands $m > 10\,{\rm TeV}$.

2.4 The assumption of randomization

In the curvaton scenario, the curvature perturbation ζ is given up to possible post-inflationary evolution by eq. (2.11), involving the fractional perturbation $(\delta\sigma/\sigma)_*$ of the curvaton field a few Hubble times after horizon exit during inflation. The perturbation $\delta\sigma$ is completely under control, being a scale-invariant gaussian quantity with the famous spectrum $(H_*/2\pi)^2$. To calculate the *fractional* perturbation though, one needs the unperturbed value σ_*. It represents the average value of σ within the comoving region that will become our observable Universe, evaluated a few Hubble times after that region leaves the horizon. The average of σ, within a region of fixed size H_*^{-1}, evolves under the combined effects of the classical slow roll and the quantum fluctuation. In one Hubble time, the classical slow roll gives a change $\Delta\phi = -V'/H_*^2$, while the quantum fluctuation gives a random contribution $\Delta\phi = \pm H_*/2\pi$. The classical motion dominates if $|V'| \gg H_*^3$, but even if this condition holds early on it cannot be satisfied indefinitely because the classical motion of σ will always be towards a minimum where V' vanishes.

We shall therefore assume[4] that before our Universe leaves the horizon the quantum fluctuation has come to dominate, placing σ_* within a region around the minimum given by $|V'| \lesssim H_*^3$. Remembering that the maximum value of σ is πv, and using the approximation $V \simeq \frac{1}{2}m^2\sigma^2$, one can see that this region corresponds to

$$\sigma_* \lesssim \min\left\{\frac{H_*^3}{m^2}, v\right\}. \tag{2.19}$$

From these considerations, we see that there is no precise prediction for the spectrum of the observed curvature perturbation in the curvaton scenario. Rather, the spectrum in the curvaton scenario is itself a stochastic quantity, depending on the location of our Universe within the much larger perturbed region that presumably surrounds it. This state of affairs arises because the spectrum, as far as observation is concerned, should not be defined within an arbitrarily large region around us. Instead, it should be defined in a

[4] In [28] it was necessary to make the opposite assumption, in order to obtain a sufficiently big curvature perturbation.

JHEP07(2003)053

region around us only a few orders of magnitude bigger than the one we observe. Up to a numerical factor of order unity, the spectrum is simply the mean-square value of the curvature perturbation is such a region.[5]

If it is assumed that we live in a typical region, then the order of magnitude of σ_* is predicted to be the right hand side of eq. (2.19). In that case, the order of magnitude of the value of the spectrum (given by eq. (2.13)) is also predicted. The spectrum in other locations is then typically about the same as in the our region, and is otherwise much *bigger*.

If instead it is assumed that we live in a region where σ_* is many orders of magnitude smaller than the typical value, then there are regions where the spectrum of the curvature perturbation is both much bigger and much smaller. In that case, the fact that the spectrum has roughly the observed value would presumably be the result of anthropic selection [50]. Which of these two assumptions is observationally viable depends on the model.

2.5 Inequalities following from the randomization assumption

Using eq. (2.14), one can write eq. (2.19) as

$$\left(\frac{r}{A}\right)^{\frac{1}{2}} m \lesssim H_* \lesssim \left(\frac{A}{r}\right) v \,, \tag{2.20}$$

where $A = 5 \times 10^{-4}$. The consistency of the two inequalities requires

$$\frac{m}{v} \lesssim 10^{-2} \left(\frac{10^{-2}}{r}\right)^{\frac{3}{2}} < 10^{-2} \,. \tag{2.21}$$

If we live in a typical location, H_* will approximately saturate either the upper or the lower limit of eq. (2.20). Allowing r to vary over its range $10^{-2} \lesssim r < 1$ merges these possibilities if

$$\frac{m}{v} \gtrsim A^{\frac{3}{2}} \sim 10^{-5} \,. \tag{2.22}$$

Otherwise they represent two distinct ranges of H_*, with a gap between them.

The first inequality of eq. (2.20) leads to

$$vm \lesssim 0.2 v H_* \,. \tag{2.23}$$

Provided that $v \lesssim M_{\rm P}$, this guarantees that the curvaton potential is negligible during inflation (eq. (2.4)).

Another way of writing the first inequality is

$$\frac{m^2}{H_*^2} \lesssim 0.05 \left(\frac{10^{-2}}{r}\right) < 0.05 \,. \tag{2.24}$$

This is particularly interesting, because the spectral index in the curvaton scenario is given by [4]

$$n = 1 + \frac{2}{3}\frac{V''(\sigma_*)}{H_*^2} - 2\epsilon_H \,, \tag{2.25}$$

[5] An analogous situation is encountered for the axion isocurvature perturbation irrespective of the scenario for the generation of the curvature perturbation [44].

JHEP07(2003)053

where $\epsilon_H \equiv |\dot{H}/H^2|_*$. It is expected that PLANCK will measure n to an accuracy ± 0.01, which is therefore the target accuracy for theoretical predictions. Almost all inflation models give $\epsilon_H \ll 0.01$ so that this term can be ignored, and eq. (2.24) shows that the other term is also negligible unless r is close to saturating its present bound. We conclude that *the spectral index is almost certainly indistinguishable from* 1 *if the curvaton is a PNGB*. This is line with the general expectation for the curvaton scenario [12], coming from the fact that in contrast with the inflaton, the curvaton does not know about the end of inflation.

Finally, we note that the second inequality of eq. (2.20) with eq. (2.17) leads to

$$\frac{v}{M_{\rm P}} \gtrsim 10^{-1} \left(\frac{\Gamma}{m}\right)^{1/4}. \tag{2.26}$$

In a particular model, one should look out for a possible conflict between this bound and eq. (2.28).

2.6 Implementing the estimates

For a PNGB to be a viable curvaton candidate, its parameters must satisfy the inequalities eqs. (2.18), (2.21) and (2.26), which we repeat here for convenience:

$$\Gamma \gtrsim 4.5 \times 10^{-25}\text{GeV} \tag{2.27}$$

$$\frac{m}{v} \lesssim 10^{-2} \left(\frac{10^{-2}}{r}\right)^{3/2} < 10^{-2} \tag{2.28}$$

$$\frac{v}{M_{\rm P}} \gtrsim 10^{-1} \left(\frac{\Gamma}{m}\right)^{1/4}. \tag{2.29}$$

In each case, we will check that these requirements can be satisfied.

In addition, the scale of inflation H_* must lie within the range eq. (2.20) and must satisfy eq. (2.17). We consider these bounds though only if they are of particular interest for the curvaton candidate itself. The same goes for eq. (2.16), which in principle determines the parameter r if we know enough about the cosmology.

3. The symmetry broken by non-renormalizable terms

A simple possibility, mentioned already in [4], is that the symmetry is broken mainly by non-renormalizable terms. Such a situation has been widely discussed in the case of the Peccei-Quinn symmetry, where it would be a disaster since the breaking in that case is suppose to be through QCD instantons. Following [4], suppose that only one field Ψ spontaneously breaks the symmetry, and only a single non-renormalizable term explicitly breaks it. Keeping only that term, the potential of Ψ is

$$V(\Sigma) = V_0 - m_\Sigma^2 |\Sigma|^2 + \lambda |\Sigma|^4 + \lambda_d \left(\frac{\Sigma^d}{M_{\rm P}^{d-4}} + \text{c.c.}\right), \tag{3.1}$$

JHEP07(2003)053

where $\lambda_d \sim 1$ on the assumption that the ultra-violet cutoff is at the Planck scale. This generates a VEV $\langle|\Psi|\rangle \equiv \sqrt{2}v$, in which the canonically-normalized PNGB σ defined by $\Sigma = \sqrt{2}ve^{i\sigma/v}$ has the potential eq. (2.1) with

$$m^2 \sim M_{\rm P}^2 \left(\frac{v}{M_{\rm P}}\right)^{d-2} . \tag{3.2}$$

This estimate of m will remain valid if other fields spontaneously break the symmetry, provided that their VEV's are of similar magnitude. Using it, the bound eq. (2.28) becomes

$$\frac{v}{M_{\rm P}} \lesssim 10^{-\frac{4}{d-4}} . \tag{3.3}$$

To evaluate the other bound eq. (2.29) we need an estimate of the curvaton decay rate. If the decay proceeds through dimensionless couplings of Σ that are of order 1, the rate will be $\Gamma \sim m^3/v^2$. Then the bound eq. (2.29) becomes

$$\left(\frac{v}{M_{\rm P}}\right)^{8-d} > 10^{-4} . \tag{3.4}$$

It contradicts eq. (3.3) for $d = 5$ and 6, but is compatible for $d = 7$ and trivial for higher values. If instead the curvaton decay is of gravitational strength, $\Gamma \sim m^3/M_{\rm P}^2$ and then eq. (2.29) becomes

$$\left(\frac{v}{M_{\rm P}}\right)^{6-d} > 10^{-4} . \tag{3.5}$$

This contradicts eq. (3.3) for $d = 5$ and is trivial for higher values. Subject to these restrictions, this PNGB model seems to satisfy all of the requirements.

4. The curvaton as a Wilson line

In this section we discuss the possibility that the curvaton is identified with the extra-component of a gauge field in a compactified five-dimensional theory. Spurred by similar recent considerations applied to models of inflation [41, 42, 43] and quintessence [51], we consider a five-dimensional model with the extra fifth dimension compactified on a circle of radius R and identify the curvaton with the fifth component A_5 of an abelian gauge field A_M ($M = 0, 1, 2, 3, 5$) propagating in the bulk (the generalization to the non-abelian case is straightforward). As such, the curvaton field cannot have a local potential because of the higher-dimensional gauge invariance. However, a non-local potential as a function of the gauge-invariant Wilson line

$$e^{i\theta} = e^{i \oint g_5 A_5 \, dy} , \tag{4.1}$$

where y is the coordinate along the fifth dimension, $0 \leq y < 2\pi R$, will be generated in the presence of fields charged under the abelian symmetry [52]

Writing the field A_5 as

$$A_5 = \frac{\theta}{2\pi g_5 R} , \tag{4.2}$$

JHEP07(2003)053

where g_5 is the five-dimensional gauge coupling constant, at energies below the scale $1/R$, and θ looks like a four-dimensional field with lagrangian

$$\mathcal{L} = \frac{1}{2g_4^2(2\pi R)^2}\,(\partial_\mu\theta)^2 - V(\theta)\,, \tag{4.3}$$

where $g_4 = g_5/(2\pi R)^{1/2}$ is the four-dimensional gauge coupling constant. The canonically normalized field is $\sigma = v\theta$, with

$$v = \frac{1}{2\pi g_4 R}\,. \tag{4.4}$$

The higher-dimensional nature of the theory preserves the curvaton potential from acquiring dangerous corrections, and non-local effects must be necessarily exponentially suppressed because the typical length of five-dimensional quantum gravity effects $\sim M_5^{-1}$, where M_5 is the five-dimensional Planck scale, is much smaller than the size of the extra-dimensions. The global nature of the Wilson line preserves its potential from acquiring large ultraviolet (local) corrections of the form $\Lambda_{\rm UV}^2\sigma^2$ — where $\Lambda_{\rm UV}$ is the ultraviolet cut-off of the theory — and therefore the flatness of the potential is preserved. This makes the Wilson line a perfect candidate for a curvaton field.

Let us now turn to the form of the curvaton potential. We assume that the potential for the curvaton field is generated radiatively by a set of bulk fields which are charged under the U(1) symmetry with charges q_a. From the four-dimensional point of view, this is equivalent to having a tower of Kaluza-Klein states with squared masses

$$m_a^2 = \left(\frac{n}{R} + g_4\,q_a\,\sigma\right)^2, \qquad (n = 0, \pm1, \pm2, \ldots)\,. \tag{4.5}$$

Borrowing from finite temperature field theory calculations, the potential can be written as [53]

$$V(\sigma) = \frac{1}{128\pi^6 R^4}\,{\rm Tr}\left[V\left(r_a^F, \sigma\right) - V\left(r_a^B, \sigma\right)\right], \tag{4.6}$$

where the trace is over the number of degrees of freedom, the superscripts F and B stand for fermions and bosons, respectively and

$$V\left(r_a, \sigma\right) = 3\,{\rm Li}_5\left(r_a\right) + {\rm h.c.}\,. \tag{4.7}$$

We have defined

$$r_a = e^{iq_a\sigma/v}\,, \tag{4.8}$$

and in eq. (4.7) the functions ${\rm Li}_n(z)$ stand for the polylogarithm functions

$${\rm Li}_n(z) = \sum_{k=1}^{\infty}\frac{z^k}{k^n}\,. \tag{4.9}$$

The potential (4.7) is well approximated by the form (2.1) with

$$V_0 \simeq \frac{3c}{16\pi^6}\frac{1}{R^4}\,, \tag{4.10}$$

JHEP07(2003)053

where $c \sim 1$ is a numerical coefficient depending upon the charges of the bulk fields. With eqs. (2.1) and (4.4) this corresponds to mass

$$m \sim \frac{10^{-1} g_4}{R} \,. \tag{4.11}$$

The curvaton decays into the zero modes of the bulk fields, with rate

$$\Gamma \sim 10^{-1}\, g_4^2\, m \sim 10^{-2}\, \frac{g_4^3}{R} \,. \tag{4.12}$$

The conditions eqs. (2.27), (2.28) and (2.29) give respectively

$$g_4 \gtrsim 10^{-13} (M_{\rm P} R)^{1/3} \tag{4.13}$$

$$g_4 \lesssim 10^{-1} \tag{4.14}$$

$$g_4 \lesssim (M_{\rm P} R)^{-2/3} \,. \tag{4.15}$$

The first and third of these together lead to the weak constraint $R^{-1} \gtrsim 10^5$ GeV.

Since the effective four-dimensional field theory under consideration is supposed to be valid during inflation, we require $H_* R \ll 1$, and this leads to two more constraints on the curvaton parameters. One, corresponding to the first bound in eq. (2.20) is automatic by virtue of eq. (4.14). The other, corresponding to eq. (2.17), is

$$g_4 \lesssim 10^7 (M_{\rm P} R)^{-2} \,, \tag{4.16}$$

which with eq. (4.13) gives

$$\frac{1}{R} \gtrsim 10^{10}\ {\rm GeV} \,. \tag{4.17}$$

The constraints follow just from the requirement that inflation lasts long enough for the curvaton to be somewhere in the quantum regime. The most important results are the lower bound eq. (4.17) on the size of the extra dimension, and eq. (4.16) which requires a very small coupling g_4 unless the extra dimension is quite small.

In the scenario where the curvaton is identified with the Wilson line of an extra-dimensional gauge field, the typical scale σ_* is expected to be of the order of $v = \frac{1}{2\pi g_4 R}$ in the randomized case. This happens if $H_*^3/m^2 \lesssim v$. Using eq. (2.14), one easily finds that the randomized case is attained if

$$g_4 \gtrsim \left(\frac{A}{r}\right)^{3/4} \,, \tag{4.18}$$

which imposes $g_4 \gtrsim 10^{-1}$ for $r \simeq 10^{-2}$ and $g_4 \gtrsim 10^{-3}$ for $r \simeq 1$. The mass during inflation is expected to be $m^2 = V'' \sim V_0/v^2 \sim 10^{-3} g_4^2/R^2$.

In order to understand for which values of the parameters R and g_4 the Wilson line can be a curvaton field, we have to take into account as well the conditions

$$m \lesssim H_* \lesssim R^{-1} \tag{4.19}$$

JHEP07(2003)053

which, expressed in terms of the parameter r, give

$$\frac{5\times 10^{-4}}{g_4} \lesssim r \lesssim \frac{10^{-3}}{g_4^2}\,. \tag{4.20}$$

The fact that the parameter r is constrained by the WMAP bound on non-gaussianity to be between 10^{-2} and 1, puts severe constraints on g_4. For example, if we want $r = 10^{-2}$ we should require the extreme fine-tuning $5\times 10^{-2} \lesssim g_4 \lesssim 3\times 10^{-1}$ which, together the randomization condition does not leave any room left. The situation improves if we require that r is close to unity. In such a case, taking into account the randomization condition, one has more room, $10^{-3} \lesssim g_4 \lesssim 3\times 10^{-2}$. Therefore, in scenarios where the curvaton is a Wilson line it seems more natural that the curvaton decays after it dominates the energy density of the Universe. From what is discussed in 2.3 this makes the spectral index n indistinguishable from 1. Imposing that this is the case gives

$$10^{-2}\,\frac{g_4^3}{R} \lesssim m\,\frac{\sigma_*^4}{M_{\rm P}^4}\,\min\left\{1, \frac{H_{\rm REH}}{m}\right\}. \tag{4.21}$$

If $H_{\rm REH} \gtrsim m$, then eq. (4.21) gives

$$\frac{1}{R} \gtrsim 10\, g_4^{3/2}\, M_{\rm P} \gtrsim 10^{-5}\, M_{\rm P} = 10^{14}\,{\rm GeV}\,, \tag{4.22}$$

where in the last passage we have made use of the fact that $10^{-3} \lesssim g_4$. This result is correct as long as $H_{\rm REH} \gtrsim m \sim 10^{-1} g_4 R^{-1} \gtrsim g_4^{5/2}\, M_{\rm P} \sim 10^{10}$ GeV.

If $H_{\rm REH} \lesssim m$, then eq. (4.21) gives

$$g_4^7\, R^3\, M_{\rm P}^4 \lesssim 10^{-2}\, H_{\rm REH} \lesssim 10^{-2}\, m\,, \tag{4.23}$$

which gives again condition (4.22) on the scale R^{-1}. We conclude that the scenario in which the curvaton is identified with a Wilson line the amount of non-gaussianity produced in the cosmological perturbations is minimal and the scale of the extra-dimension has to be smaller than $10^{-14}\,{\rm GeV}^{-1}$.

The non-randomized case, $H_*^3/m^2 \gtrsim v$, is attained if, using eq. (2.14),

$$g_4 \lesssim \left(\frac{A}{r}\right)^{3/4}\,, \tag{4.24}$$

which imposes $g_4 \lesssim 10^{-1}$ for $r \simeq 10^{-2}$ and $g_4 \lesssim 10^{-3}$ for $r \simeq 1$. Again, we have to take into account the conditions $m \lesssim H_* \lesssim R^{-1}$ which are converted into $\frac{5\times 10^{-4}}{g_4} \lesssim r \lesssim \frac{10^{-3}}{g_4^2}$. If we impose $r = 10^{-2}$ we should require some fine-tuning $5\times 10^{-2} \lesssim g_4 \lesssim 10^{-1}$. The situation slightly improves if we require that r is close to unity. In such a case, taking into account the non-randomization condition, one has more room, $10^{-4} \lesssim g_4 \lesssim 10^{-3}$. Again, in scenarios where the curvaton is a Wilson line and the randomized case is not attained, it seems more natural that the curvaton decays after it dominates the energy density of the Universe. From what is discussed in 2.3 this makes the spectral index n indistinguishable from 1. Imposing that this is the case gives

$$10^{-2}\,\frac{g_4^3}{R} \lesssim m\,\frac{\sigma_*^4}{M_{\rm P}^4}\,\min\left\{1, \frac{H_{\rm REH}}{m}\right\}. \tag{4.25}$$

JHEP07(2003)053

If $H_{\rm REH} \gtrsim m$, then eq. (4.25) gives

$$\frac{1}{R} \gtrsim 10\, g_4^{3/2}\, M_{\rm P} \gtrsim 10^{-5}\, M_{\rm P} = 10^{13}\ {\rm GeV}, \tag{4.26}$$

where in the last passage we have made use of the fact that $10^{-4} \lesssim g_4$. This result is correct as long as $H_{\rm REH} \gtrsim m \sim 10^{-1} g_4 R^{-1} \gtrsim g_4^{5/2}\, M_{\rm P} \sim 10^{10}$ GeV.

If $H_{\rm REH} \lesssim m$, then eq. (4.25) gives

$$g_4^7\, R^3\, M_{\rm P}^4 \lesssim 10^{-2}\, H_{\rm REH} \lesssim 10^{-2}\, m\,, \tag{4.27}$$

which gives again condition (4.26) on the scale R^{-1}. We conclude that the scenario in which the curvaton is identified with a Wilson line the amount of non-gaussianity produced in the cosmological perturbations is minimal and the scale of the extra-dimension has to be smaller than $10^{-13}\ {\rm GeV}^{-1}$.

5. The curvaton as a Little Higgs

Little Higgs (LH) theories [54] are theories in which the mass of the scalar field σ (the Higgs) is stabilized against radiative corrections by making the scalar field a PNGB resulting from a spontaneously broken (approximate) symmetry $\mathcal{G}$ at some scale f. In order to generate a potential for σ one needs to explicitly break the initial global symmetry. The novel feature is that instead of breaking the initial symmetry with a single coupling, one introduces two couplings such that each coupling by itself preserves sufficient amount of symmetry to guarantee the masslessness of the PNGB. Schematically, to the initial lagrangian $\mathcal{L}_0$ one adds two terms $(\mathcal{L}_1 + \mathcal{L}_2)$ with couplings g_1 and g_2 respectively and each term is chosen such that by itself it preserves a different subset of global symmetries under which the σ is an exact Nambu-Goldstone boson. The one-loop mass of σ is then necessarily proportional to the product of $g_1 g_2$

$$m^2 \simeq c\, \frac{g_1^2 g_2^2}{16\pi^2}\, f^2\,. \tag{5.1}$$

where c is a coefficient of order unity which we shall assume is positive. The Little Higgs potential is not necessarily sinusoidal, but it is periodic with periodic of order f. To obtain order of magnitude estimates we therefore set $v \sim f$. The estimates will be valid provided that the curvaton field during inflation is small enough that the quadratic approximation eq. (2.2) is roughly valid.

Taking for simplicity $g_1 \sim g_2 \sim g$, the decay rate of the curvaton is given by

$$\Gamma \sim 10^{-1} \frac{m^3}{f^2} \sim 10^{-4}\, g^6\, f\,. \tag{5.2}$$

The conditions eqs. (2.27), (2.28) and (2.29) give respectively

$$g^6 f \gtrsim 10^{-20}\ {\rm GeV} \tag{5.3}$$

$$g^2 \lesssim 10^{-1} \tag{5.4}$$

$$g \lesssim 10^2\, \frac{f}{M_{\rm P}} \tag{5.5}$$

JHEP07(2003)053

The consistency of the first condition with the third places a lower bound on the scale of the Little Higgs,

$$f \gtrsim 10^{11}\,\mathrm{GeV}\,, \tag{5.6}$$

and the third condition itself requires that this scale should be rather high if the coupling g is not to be very small. Subject to these conditions, a Little Higgs seems to be a viable curvaton candidate.

6. The curvaton as a string axion

Another interesting possibility is considering the so-called string axion fields as potential curvatons. The string axions are the imaginary parts of the string moduli fields, which correspond to flat directions that are not lifted by supergravity corrections. Moduli fields are a necessary ingredient of string theories. Their Kähler potential at tree level is

$$K = -M_P^2 \sum_i \ln\left[\frac{S_i + \bar{S}_i}{M_\mathrm{P}}\right], \tag{6.1}$$

which is independent from $\mathrm{Im}\, S_i$. Moreover, even though the superpotential for the moduli may receive a non-perturbative contribution of the form

$$W \simeq \sum_j \Lambda_j^3 \exp\left(-\frac{\sum_i \beta_{ij} S_i}{M_\mathrm{P}}\right) \tag{6.2}$$

the F-term scalar potential V_F (which is only a function of K and W and their derivatives) may remain flat in the $\mathrm{Im}\, S_i$ directions because, in general, W is dominated by the term with the largest Λ_j, in which case V_F is independent of the phase of W. The $\mathrm{Im}\, S_i$ satisfy an additional discrete symmetry

$$\mathrm{Im}\, S_i = \mathrm{Im}\, S_i + 2\pi v\,, \tag{6.3}$$

where, $v \lesssim M_\mathrm{P}$. The string axions of 4-dimensional supergravity are associated with the components of the antisymmetric tensor B of string theory [55]

$$B = b_{\mu\nu} dx^\mu dx^\nu + b_I \omega^I_{\alpha\beta} dy^\alpha dy^\beta\,, \tag{6.4}$$

where $\omega^I_{\alpha\beta}$ represents the topology of the compactified space corresponding to the extra dimensions y^α. The so-called model-independent string axion $\mathrm{Im}\, S$ is related to the four-dimensional spacetime components $b_{\mu\nu}$, whereas the model-dependent string axions $\mathrm{Im}\, T_i$ are related to b_I, which depends on the compactification. Their respective real parts are the dilaton $\mathrm{Re}\, S = 4\pi/g^2_\mathrm{GUT}$ and the so-called geometrical moduli $\mathrm{Re}\, T_i$, associated to the volume of the extra dimensions. Thus, string theory provides us with natural candidates for the curvaton since the string axions are PNGBs, whose potential appears due to SUSY breaking both during and after inflation.

In contrast to the other PNGB cases in the case of the string axion one has to worry for possible corrections to the potential due to SUSY breaking during inflation. In view

JHEP07(2003)053

of eqs. (6.1) and (6.2) a simple calculation shows that the effective mass (due to SUSY breaking) of the canonically normalized string axion field is[6]

$$m_*^2 = V_F''(\sigma) \sim \sqrt{V_*}\left(\frac{\Lambda}{M_P}\right)^3, \tag{6.5}$$

where $\Lambda \equiv \max\{\Lambda_j\}$. This turns out to be much smaller than H_*^2 if $V_*^{1/4} \geq M_S$, where $M_S \sim \sqrt{\Lambda^3/M_P} \sim 10^{11}\,\mathrm{GeV}$ is the SUSY breaking scale in the vacuum (for gravity mediated SUSY breaking). Thus, during inflation the string axion is overdamped and remains effectively frozen. As discussed below, if the string axion is to be a successful curvaton, it is hard for the inflationary scale to be very low, which results in the complete randomization of the field. Thus, at the end of inflation the expected misalignment of the field is maximal, i.e. $\sigma_* \sim v$. After the end of inflation the string axion potential is not disturbed by Kähler corrections, as is the case for all PNGBs. It is reasonable to assume, then, that the potential remains negligible[7] until much later times when the string axion unfreezes and begins its oscillations. At that time the height of the potential is given (due to gaugino condensates and also by worldsheet, membrane instantons) by the scale of SUSY breaking M_S.

Now, typically, $v \sim M_{\rm P}$, which suggests that the mass of the string axion is $m \sim m_{3/2} \sim 1\,\mathrm{TeV}$. One, then, finds $\mathcal{P}_\zeta^{1/2} \sim r\mathcal{P}_{\delta\sigma/\sigma}^{1/2} \sim rH_*/M_{\rm P}$, which, according to eq. (2.14), gives $V_*^{1/4} \sim r^{-1/2}10^{16}\,\mathrm{GeV}$. However, this violates the bound of eq. (2.3). As a result the contribution of the inflaton's curvature perturbation is not negligible, in contrast to what is usually assumed under the curvaton hypothesis. Thus, we see that *generically the string axions contribute substantially to the curvature perturbation* even under the inflaton hypothesis. This may generate important isocurvature perturbations because the inflaton and the string axion perturbations are uncorrelated. Also, since $v \sim M_P$, the spectral index of the curvaton perturbations is $n \approx 1$. Therefore, we expect the spectral index of the overall spectrum to be given by the inflaton perturbations on the large (small) scales and by the string axion perturbations on small (large) scales if the inflaton's perturbation spectrum is red (blue). The switch-over scale is model dependent and is determined by the relative importance of the contribution to the curvature perturbations coming from the inflaton and the string axions respectively. Note that, since $m \sim 1\,\mathrm{TeV}$ the string axions violate the BBN constraint eq. (2.27). This is the well known moduli problem, which, however, can be overcome using one of the mechanisms that string theory assumes, e.g. a brief period of thermal inflation.

The above scenario is interesting in its own right but it does not benefit from the liberation effects of the curvaton hypothesis to inflationary model building, since the inflaton's curvature perturbations are also important. Fortunately, many string theory models offer alternative possibilities. In particular, it is possible to have $v < M_{\rm P}$ [58, 59, 60]. Furthermore, strongly coupled string theory allows for large values of the dilaton and the geometric moduli, in which case the scale of V_F is suppressed as [60, 61, 62] $(vm)^2 \sim e^{-2\tau}M_S^4$, where $\tau \equiv \pi\,\mathrm{Re}T/M_{\rm P}$ parametrizes the overall modulus T.

[6] This result holds true even when W is dominated by more than one terms of the sum in eq. (6.2), as is the case e.g. of multiple gaugino condensates [56] (racetrack scenario [57]).

[7] Even though it may be possibly modified in early times by the evolution of $\mathrm{Re}T$, while $\mathrm{Im}T$ is frozen.

JHEP07(2003)053

Let us investigate the situation by taking[8]

$$\sigma_* \sim v \equiv \alpha M_{\rm P} \quad \Rightarrow \quad m \sim \alpha^{-1} e^{-\tau} m_{3/2}\,, \tag{6.6}$$

where $\alpha \leq 1$ and for gravitational decay of the string axion we will use $\Gamma \sim m^3/M_{\rm P}^2$. The first requirement is that the decay of the string axion occurs before BBN. In view of eq. (2.27), this requirement results in the bound

$$\alpha < 0.1\, e^{-\tau}\,. \tag{6.7}$$

Now, it is easy to see that the COBE requirement (c.f. eq. (2.14)), in this case, demands

$$V_*^{1/4} \sim \sqrt{\alpha}\, M_{\rm P} \mathcal{P}_\zeta^{\frac{1}{4}} r^{-1/2}\,. \tag{6.8}$$

The dynamics of the string axion depend on whether the field begins to oscillate after the onset of radiation domination or not. Comparing $H_{\rm REH}$ with $H_{\rm OSC} \sim m$ we find that

$$H_{\rm REH} \gtrless m \quad \Leftrightarrow \quad \alpha \gtrless e^{-\tau}\, 10^3 \gamma^{-1}\,, \tag{6.9}$$

where we defined

$$\gamma \equiv \frac{H_{\rm REH}}{1\ {\rm GeV}} > 10^{-24} \tag{6.10}$$

and the lower bound is due to the requirement that the radiation era begins before BBN, i.e. $H_{\rm BBN} < H_{\rm REH}$.

From eq. (6.9) and in view of the constraint (6.7), we see that BBN demands that the string axion begins oscillating before reheating if $\gamma < 10^4$. In the case where the radiation era begins after the inflationary reheating we have $H_{\rm REH} \sim \Gamma_\Phi$, where Γ_Φ is the decay rate of the inflaton field. Then, the gravitino bound on the reheating temperature $T_{\rm REH} \sim \sqrt{\Gamma_\Phi M_P} \leq 10^9\,{\rm GeV}$ suggests that $\gamma \leq 1$. However, if the string axion dominates the Universe before it decays the gravitino bound may be substantially relaxed by the subsequent entropy production [58] and we may have $\gamma \gg 1$ without problem. Furthermore, even if the string axion decays before it dominates the Universe, one may still have $\gamma \gg 1$ if extra entropy production occurred during the period between the inflationary reheating and the field's decay. This may well be possible if, during this period, there was a brief period of thermal inflation or the Universe was dominated by another oscillating field, whose curvature perturbation (like the inflaton's) is also negligible. Due to the above we will treat γ as a free parameter.

6.1 Onset of curvaton oscillations before radiation domination ($H_{\rm REH} < m$)

Case A: consider firstly that the curvaton decays before dominating. In this case, from eq. (2.16) and using eq. (6.6) we have

$$r \sim \alpha^{7/2} e^{3\tau/2} 10^{14} \gamma^{1/2}\,. \tag{6.11}$$

[8] Note that the effective mass m_* of the string axion during inflation is also suppressed by $e^{-\tau}$.

JHEP07(2003)053

Then eq. (6.8) becomes

$$V_*^{1/4} \sim \alpha^{-5/4} e^{-3\tau/4} 10^9 \gamma^{-1/4} \mathrm{GeV}\,. \tag{6.12}$$

Using the WMAP range $10^{-2} \leq r < 1$, eq. (6.11) gives

$$e^{-3\tau/7} 10^{-5} \gamma^{-1/7} \leq \alpha < \min\left\{e^{-3\tau/7} 10^{-4} \gamma^{-1/7},\, e^{-\tau} 0.1\right\}, \tag{6.13}$$

where we also took eq. (6.7) into account. In view of the above, eq. (6.12) becomes

$$\max\left\{e^{-3\tau/14} 10^{14} \gamma^{-1/14}, e^{\tau/2} 10^{10} \gamma^{-1/4}\right\} \mathrm{GeV} < V_*^{1/4} \leq e^{-3\tau/14} 10^{15} \gamma^{-1/14} \mathrm{GeV}\,, \tag{6.14}$$

where the second element in the brackets is due to the BBN constraint.

Case B: now consider that the curvaton dominates before decaying. In this case, since $r \sim 1$, eq. (6.8) is simply

$$V_*^{1/4} \sim \sqrt{\alpha}\, 10^{16} \mathrm{GeV}\,. \tag{6.15}$$

Hence, in view of eq. (6.7) and also demanding that $\Gamma < H_{\mathrm{DOM}}$, we find

$$e^{-3\tau/7} 10^{-4} \gamma^{-1/7} \leq \alpha < e^{-\tau} 0.1\,, \tag{6.16}$$

through which eq. (6.15) is recast as

$$e^{-3\tau/14} 10^{14} \gamma^{-1/14} \mathrm{GeV} \leq V_*^{1/4} < e^{-\tau/2} 10^{16} \mathrm{GeV}\,. \tag{6.17}$$

6.2 Onset of curvaton oscillations after radiation domination ($H_{\mathrm{REH}} \geq m$)

Case A: consider that the curvaton decays before dominating. In this case eqs. (2.16) and (6.6) give

$$r \sim \alpha^3 e^{\tau} 10^{15}\,. \tag{6.18}$$

Then, from eqs. (6.8) and (6.18), we find:

$$V_*^{1/4} \sim \alpha^{-1} e^{-\tau/2} 10^9 \mathrm{GeV}\,. \tag{6.19}$$

Employing again the WMAP range $10^{-2} \leq r < 1$, eq. (6.18) gives

$$e^{-\tau/3} 10^{-6} \leq \alpha < \min\left\{e^{-\tau/3} 10^{-5},\, e^{-\tau} 0.1\right\}, \tag{6.20}$$

where we also took eq. (6.7) into account. Using the above in eq. (6.19) we obtain:

$$\max\left\{e^{-\tau/6} 10^{14}, e^{\tau/2} 10^{10}\right\} \mathrm{GeV} < V_*^{1/4} \leq e^{-\tau/6} 10^{15}\,, \tag{6.21}$$

where the second element in the brackets is due to the BBN constraint.

JHEP07(2003)053

Case B: now consider that the curvaton dominates before decaying. In this case eq. (6.8) suggests that $V_*^{1/4}$ is again given by eq. (6.15). Using the BBN constraint (6.7) and also demanding that $\Gamma < H_{\rm DOM}$, we find

$$e^{-\tau/3}10^{-5} \leq \alpha < e^{-\tau}0.1\,, \tag{6.22}$$

through which eq. (6.15) is recast as

$$e^{-\tau/6}10^{14}{\rm GeV} \leq V_*^{1/4} < e^{-\tau/2}10^{16}{\rm GeV}\,. \tag{6.23}$$

The above show that as α grows the string axion decays later, because $\Gamma \propto m^3 \propto \alpha^{-3}$. However, eqs. (6.12), (6.19) and (6.15) show that, for growing α, the energy scale of inflation decreases if the string axion decays before domination (Case A) but increases instead if the latter decays after it dominates the Universe (Case B). Thus, the minimum possible V_* (for a given τ) occurs when the decay of the string axion takes place at just about the time when its density comes to dominate the Universe. This value is

$$(V_*^{1/4})_{\rm min} \sim \begin{cases} e^{-3\tau/14}10^{14}\gamma^{-1/14}\,{\rm GeV} & H_{\rm REH} < m \\ e^{-\tau/6}10^{14}\,{\rm GeV} & H_{\rm REH} \geq m \end{cases}\,. \tag{6.24}$$

In the case when the oscillations begin before (after) the onset of radiation domination, from eq. (6.16) (eq. (6.22)) we see that, it is acceptable for the string axion to decay after domination (Case B) only if $\tau < 12 + \frac{1}{4}\ln\gamma$ ($\tau < 14$). In this range the BBN constraint is subdominant in the case when the string axion decays before domination (Case A). For larger τ however, it is the BBN constraint that sets the bounds in eqs. (6.13) and (6.14) (eqs. (6.20) and (6.21)). Still, τ cannot grow much larger. Indeed, the parameter space for subdominant string axion decay disappears too when $\tau \to 16 + \frac{1}{4}\ln\gamma$ ($\tau \to 17$), which is much smaller than the requirements of strongly coupled heterotic string theory [60, 61]: $\tau \lesssim \pi/\alpha_{\rm GUT} \approx 79$ even if γ is large.[9] However, the parameter space can be enlarged toward larger values of τ if the BBN constraint is relaxed, which may be possible if one considers say a brief period of thermal inflation subsequent to the string axion decay.

Just to get a feeling for our results let us choose $\alpha \sim 10^{-2}$ so that $v \sim 10^{16}$GeV as suggested also in refs. [59, 60]. Enforcing this into the above one finds that the string axion can indeed act as a curvaton but only if we consider weakly coupled string theory, where $e^{\tau} \sim 1$ (mainly due to the BBN constraint). Then, for $\gamma < 10^5$, the string axion begins oscillating before the onset of radiation domination. If $\gamma < 10^{-14}$ the field decays before it dominates the Universe (Case A), in which case the COBE requirement demands $V_*^{1/4} \sim 10^{11}\gamma^{-1/4}$GeV. If, however, $10^{-14} \leq \gamma < 10^5$ the field manages to dominate the Universe before decaying (Case B) and $V_*^{1/4} \sim 10^{15}$GeV. For $\gamma \geq 10^5$ the oscillations of the string axion begin after the onset of radiation domination. In this case the curvaton requirements are impossible to satisfy if the string axion decays before domination (Case A) and only the case when the string axion dominates the Universe (Case B) is allowed,

[9] The maximum value of γ corresponds to prompt inflationary reheating with $V_*^{1/4} \sim 10^{16}$GeV, which gives $\gamma_{\rm max} \sim 10^{14}$ and, therefore, $\tau_{\rm max} \simeq 24$.

JHEP07(2003)053

which, again, demands $V_*^{1/4} \sim 10^{15}$GeV. Thus, we see that with $\alpha \sim 10^{-2}$ it is marginally possible to liberate inflation from the COBE constraint (c.f. eq. (2.3)). Better results are obtained for smaller values of α, which also ensure the protection of the flatness of the potential against quantum gravity effects since $v \ll M_{\rm P}$. For example, for $\alpha \sim 10^{-4}$ one finds $V_*^{1/4} \sim 10^{14}$GeV with $\tau \leq 9$.

In summary we have shown that a string axion can be a successful curvaton but one needs to reduce the inflationary scale enough so that the contribution of the inflaton to the overall curvature perturbation is negligible. This is possible only if v is substantially smaller than M_P, which also increases the axion mass over 1 TeV and solves the moduli problem [58]. A value $v \sim 10^{16}$GeV or smaller is not unreasonable in some string theory models [59, 60]. The moduli problem reappears if one considers strongly coupled string theory (M-theory), where BBN is challenged again for large values of $\mathrm{Re}\, T$.

7. Conclusion

Mainly motivated by the fact that supersymmetry is badly broken during inflation and therefore of limited use in keeping scalar fields light, in this paper we have analyzed the possibility that cosmological perturbations are generated by a curvaton field and that the latter is a PNGB. In such a case, the mass of the curvaton field during inflation is protected by some symmetry and — in contrast to inflationary models where the inflaton field is a PNGB — the vanishing of the curvaton potential in the limit of exact symmetry does not pose any problem. We have given a general framework for discussing PNGB curvaton candidates, and explored different options.

We have shown that the curvaton may be identified with the fifth component of a gauge field living in five dimensions. The finiteness of the potential is provided by gauge invariance in five dimensions and no supersymmetry is required. It turns out that the size of the extra-dimension has to be smaller than about 10^8 of the planckian length scale for the scenario to be viable.

Alternatively, the curvaton mass may be kept light within Little Higgs theories in which the mass of the scalar field is stabilized against radiative corrections by making the scalar field a PNGB resulting from a spontaneously broken (approximate) symmetry $\mathcal{G}$ at some scale f. The novel feature is that instead of breaking the initial symmetry with a single coupling, one introduces two couplings such that each coupling by itself preserves sufficient amount of symmetry to guarantee the masslessness of the PNGB. We have found that the scale f must exceed 10^{11} GeV for this to work, and that to avoid a very small coupling the scale should be considerably higher.

Finally, we have thoroughly studied the case in which the curvaton is the string axion, the imaginary part of the string moduli fields corresponding to flat directions that are not lifted by supergravity corrections. In this case we have shown that supersymmetry breaking during inflation does not lift the flatness of the potential if the inflationary energy scale is larger than the scale of supersymmetry breaking in the vacuum. Then the string axion may be a successful curvaton if the order parameter v is smaller than M_P so that the inflaton's contribution to the curvature perturbations is negligible. Overall, our findings indicate

JHEP07(2003)053

that requiring that the effective curvaton mass be much less than the Hubble parameter during inflation is a severe constraint, but can be successfully satisfied if the curvaton is a PNGB.

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JHEP07(2003)053

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Erratum

In section 4, material was accidentally pasted in at the editing stage. These are the paragraphs from the one containing eq. (4.16) to the end of section 4. They should be ignored as they are not part of the paper.

JHEP07(2003)053

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